## You MAY NOT use a calculator.

Let $f$ be a function that is twice differentiable for all real numbers. The table below gives values of $f$ for selected points in the closed interval $2 \leq x \leq 13$.

| $x$ | 2 | 3 | 5 | 8 | 13 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 1 | 4 | -2 | 3 | 6 |

(a) Estimate $f^{\prime}(4)$. Show the work that leads to your answer.
(b) Evaluate $\int_{2}^{13}\left(3-5 f^{\prime}(x)\right) d x$. Show the work that leads to your answer.
(c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_{2}^{13} f(x) d x$.
(d) Suppose $f^{\prime}(5)=3$ and $f^{\prime \prime}(x)<0$ for all $x$ in the closed interval $5 \leq x \leq 8$. Use the line tangent to the graph of $f$ at $x=5$ to show that $f(7) \leq 4$. Use the secant line for the graph of $f$ on $5 \leq x \leq 8$ to show that $f(7) \geq \frac{4}{3}$.

