Advanced Placement Calculus

Precalculus Review

Functions and Their Graphs
Types of Functions–Shifting and Scaling
Trigonometry Review
Absolute Value and Inequalities

Functions and Their Graphs

1. Given $f(x) = 2x^2 + 3x - 4$ find $f(0), f(2), f\left(\sqrt{2}\right), f\left(1 + \sqrt{2}\right), f(-x), f(x+h), 2f(x),$ and $f(2x)$.

2. Given $f(x) = x - x^2$, find $\dfrac{f(x+h) - f(x)}{h}$.

3. Find the domain and range of $f(x) = 6 - 4x$ on $[-2, 3]$.

4. Find the domain and range of $f(x) = \sqrt{2x - 5}$.

5. Find the domain of $g(x) = \dfrac{x+2}{x^2-1}$.

6. Find the domain of $f(x) = \sqrt[4]{x^2 - 6x}$.

7. Find the domain of $f(x) = \sqrt{\dfrac{x}{\pi - x}}$.

8. Find the domain and range of $f(x) = \sqrt{-x}$.

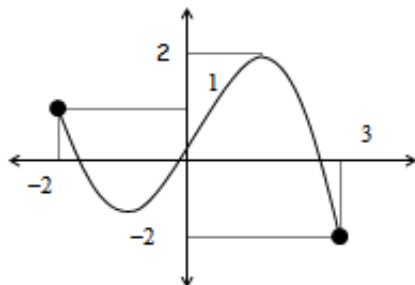9. Find the domain and range of $g(x) = \dfrac{x^2 - 1}{x - 1}$.

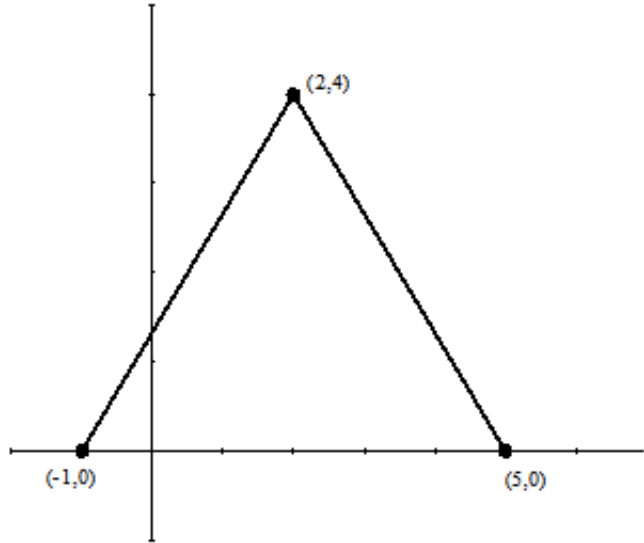10. Find the domain of $f(x) = \begin{cases} 0 & x < 2 \\ 1 & x \geq 2 \end{cases}$.

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11. Find the domain and sketch the graph of $g(x) = \begin{cases} -1 & x \leq -1 \\ 3x + 2 & |x| < 1 \\ 7 - 2x & x \geq 1 \end{cases}$.

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12. Determine if the graph below represents a function. If it does, find the domain and range.

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13. Write a function whose graph is the given curve.

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14. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 inches by 20 inches by cutting out equal squares of side x at each corner and then folding up the sides. Express the volume V of the box as a function of x.

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15. Determine if $f(x) = x^{-2}$ is odd, even, or neither.

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16. Determine if $g(x) = x^2 + x$ is odd, even, or neither.

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17. Determine if $f(x) = x^3 - x$ is odd, even, or neither.

For 18 and 19, find $f \circ g$, $g \circ f$ and their domains.

18. $f(x) = \dfrac{1}{x}$ and $g(x) = x^3 + 2x$

19. $f(x) = \sqrt[3]{x}$ and $g(x) = 1 - \sqrt{x}$

Types of Functions–Shifting and Scaling

For problems 1-4, classify each function as power, root, polynomial, rational, logarithmic, exponential, trigonometric or algebraic.

1. $f(x) = \sqrt[5]{x}$

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2. $f(x) = \dfrac{x-2}{x+6}$

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3. $f(x) = 5^x$

_____

4. $g(x) = x^4$

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5. Consider the function f. Tell how the graph of the following would change the graph of f.

   (a) $f(2x)$

   (b) $f(x-3)$

   (c) $f(x)+4$

   (d) $-f(x)$

   (e) $-f(-x)$

   (f) $4f(x)$

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6. Describe how the graph of $f(x) = -\dfrac{1}{x}$ would differ from the graph of $f(x) = \dfrac{1}{x}$.

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7. Describe how the graph of $f(x) = \dfrac{1}{x-3}$ would differ from the graph of $f(x) = \dfrac{1}{x}$.

8. Describe how the graph of $f(x) = 2 + \dfrac{1}{x-3}$ would differ from the graph of $f(x) = \dfrac{1}{x}$.

9. Describe how the graph of $f(x) = -x^2 + 2x + 1$ would differ from the graph of $f(x) = x^2$. (Hint: Complete the square to the the translated parabola in standard form.)

10. Describe how the graph of $f(x) = 2 - \sqrt{x+1}$ would differ from the graph of $f(x) = \sqrt{x}$.

11. Describe how the graph of $f(x) = 1 - (x-8)^6$ would differ from the graph of $f(x) = x^6$.

Trigonometry Review

1. Convert the following from degrees to radians:

    (a) $210°$

    (b) $900°$

    (c) $9°$

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2. Convert the following from radians to degrees:

    (a) 4π

    (b) $\dfrac{3\pi}{8}$

    (c) $\dfrac{5\pi}{12}$

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3. Find the value of all six trigonometric ratios for $\dfrac{3\pi}{4}$.

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4. Find the value of all six trigonometric ratios for $\dfrac{5\pi}{6}$.

5. Find the value of all six trigonometric ratios for $\dfrac{4\pi}{3}$.
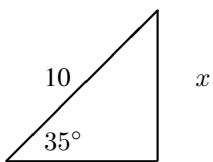
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6. Find the remaining trigonometric ratios if $\sin x = \dfrac{3}{5}$ where $x \in \left(0, \dfrac{\pi}{2}\right)$.

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7. Find the remaining trigonometric ratios if $\sec x = -\dfrac{3}{2}$ where $x \in \left(\dfrac{\pi}{2}, \pi\right)$.

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8. Find, correct to three decimals the length of the side labeled x.

Trigonometric Graphs

Determine the amplitude, period and phase-shift, the graph using the "box" method demonstrated in class.
Make sure all points are appropriately labeled.

1. $y = \sin(2x - \pi)$

2. $y = \sin \dfrac{x}{2}$

3. $y = 2\cos 2x$

4. $y = \sin(x + \pi)$

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5. $y = \cos\left(2x - \dfrac{\pi}{3}\right)$

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6. $y = \cos\left(x - \dfrac{\pi}{2}\right)$

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7. $y = 1 + \cos\left(x - \dfrac{\pi}{2}\right)$

8. $y = \sin\left(x - \dfrac{\pi}{2}\right)$

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9. $y = -1 + \sin\left(x - \dfrac{\pi}{2}\right)$

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10. $y = 1 + 2\sin\left(3x - \dfrac{\pi}{2}\right)$

Inequalities and Absolute Value

Problems 1-7, rewrite the following without absolute value.

1. $|5 - 23|$

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2. $|-\pi|$

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3. $\left|\sqrt{5} - 5\right|$

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4. $|x - 2|$ if $x < 2$

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5. $|x + 1|$

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6. $|x^2 + 1|$

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7. $|2x - 3|$

Problems 8-10, write out the meaning of the following in "plain" English.

8. $|x - 2| < 5$

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9. $|x - 3| > 3$

10. $|x + 3| < 6$

Solve the following.

11. $4x < 2x + 1 < 3x + 2$

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12. $1 - x \geq 3 - 2x > x - 6$

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13. $(x - 2)(x - 1) > 0$

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14. $2x^2 + x \leq 1$

15. $x^2 + x + 1 > 0$

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16. $x^2 \leq 3$

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17. $x^3 - x^2 \leq 0$

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18. $x^3 > x$

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19. $\dfrac{1}{x} < 4$ (Be careful! It is never a good idea to multiply both sides of an inequality by a variable!)

20. $\dfrac{4}{x} < x$

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21. $\dfrac{2x+1}{x-5} < 3$

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22. $\dfrac{x^2-1}{x^2+1} \geq 0$

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23. $|2x| = 3$

24. $|x + 3| = |2x + 1|$

25. $|x| < 3$

26. $|x - 4| < 1$

27. $|x - 5| \geq 2$

28. $|5x - 2| < 6$

29. $\left| \dfrac{x}{2 + x} \right| < 1$

30. $\left| \dfrac{2 - 3x}{1 + 2x} \right| \leq 4$