

Advanced Placement Calculus

Antidifferentiation

Antiderivatives

Rectilinear Motion

Substitution

Additional Antidifferentiation Problems

Differential Equations

Antiderivatives

1. $\int (12x^2 + 6x - 5) dx$

2. $\int (6x^9 - 4x^7 + 3x^2 + 1) dx$

3. $\int (\sqrt{x} + \sqrt[3]{x}) dx$

4. $\int \frac{6}{x^5} dx$

5. $\int \frac{t^3 + 2t^2}{\sqrt{t}} dt$

6. $\int (\sin x - 2 \cos x) dx$

7. $\int (\sec^2 t + t^2) dt$

8. $\int \frac{x^2 + x + 1}{x} dx$

9. Given $f''(x) = x^2 + x^3$, find $f(x)$. (You will have more than one constant.)

10. Given $f'(x) = 3\sqrt{x} - \frac{1}{\sqrt{x}}$ and $f(1) = 2$, find $f(x)$.

11. Given $f'(x) = \frac{2}{x}$, $x < 0$ and $f(-1) = 7$, find $f(x)$.

12. Given $f''(x) = x + \sqrt{x}$ and $f(1) = 1$ and $f'(1) = 2$, find $f(x)$.

13. Given $f'(x) = 3x^{-2}$ and $f(1) = f(-1) = 0$, find $f(x)$.

$$14. \int \frac{1}{\sqrt{1-x^2}} dx$$

$$15. \int (x + \sec x \tan x) dx$$

$$16. \int \left(\frac{3}{x^2} + \frac{5}{x^4} \right) dx$$

17. Given that the graph of f passes through the point $(1, 6)$ and that the slope of a tangent to f at any point is given by $2x + 1$, find $f(2)$.

18. $\int \frac{\sin x}{\cos^2 x} dx$

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19. The point $(3, 2)$ is on a curve and at any point (x, y) on the curve the tangent line has a slope of $2x - 3$. Find an equation of the curve.

20. At any point (x, y) on a curve, $\frac{d^2y}{dx^2} = 1 - x^2$, and an equation of the tangent line to the curve at the point $(1, 1)$ is $y = 2 - x$. Find an equation of the curve.

Rectilinear Motion

1. Given that a particle's velocity function is given by $v(t) = 3 - 2t$ and $s(0) = 4$, where s is the position function, find the position function.

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2. Given that a particle's velocity function is given by $v(t) = 3\sqrt{t}$ and $s(1) = 5$, where s is the position function, find the position function.

3. Given that a particle's acceleration function is given by $a(t) = \cos t + \sin t$ and $v(0) = 5$ and $s(0) = 0$, where s is the position function and v is the velocity function, find the position function.

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4. A stone is dropped from the upper observation deck of the CN Tower, 450 meters above the ground. Find the position function s in terms of t . Find how long it takes the stone to reach the ground and the stone's velocity on impact.

5. A ball is thrown downward from a window that is 80 feet above the ground with an initial velocity of -64 feet per second. How long will it take the ball to reach the ground and with what speed will it strike the ground?

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6. A stone is thrown vertically upward from the top of a house 60 feet above ground level with an initial velocity of 40 feet per second. How long will it take the stone to reach its maximum height? What is the maximum height? How long will it take the stone to reach the ground? With what velocity will it strike the ground?

7. Two balls are thrown upward from the edge of a cliff 423 feet above the ground. The first is thrown with a speed of 48 feet per second and the second is thrown 1 second later with a speed of 24 feet per second. Do the balls pass each other?

Antidifferentiation by Substitution

1. $\int \sqrt{3x+4} \, dx$

2. $\int x^2 (5+2x^3)^5 \, dx$

3. $\int x \cos x^2 \, dx$

4. $\int x^2 (5+2x^3)^8 \, dx$

$$5. \int \frac{4x^2}{(1-8x^3)^4} dx$$

$$6. \int x^2 \sqrt{1+x} dx$$

$$7. \int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$$

$$8. \int \sin x \sqrt{1-\cos x} dx$$

$$9. \int x\sqrt{x+2} \, dx$$

$$10. \int \frac{t}{\sqrt{t+3}} \, dt$$

$$11. \int \cos 4\beta \, d\beta$$

$$12. \int \sec^2 5x \, dx$$

$$13. \int \sin 2x \sqrt[3]{2 - \cos 2x} \, dx$$

$$14. \int \cos^2 t \sin t \, dt$$

$$15. \int (x^3 + 3)^{1/4} x^5 \, dx$$

$$16. \int \frac{4 \sin x}{(1 + \cos x)^2} \, dx$$

$$17. \int \sin \frac{1}{3}x \, dx$$

$$18. \int \frac{1}{2}t \cos 4t^2 \, dt$$

$$19. \int (x^2 - 4x + 4)^{4/3} \, dx$$

20. $\int \frac{x^2 + 2x}{\sqrt{x^3 + 3x^2 + 1}} dx$

Additional Antidifferentiation Problems

1. $\int (2x + 1)(x^2 + x + 1)^3 dx$

2. $\int e^x 2^{e^x} dx$

3. $\int r^2 \sec^2 r^3 dr$

$$4. \int \frac{1+x^3}{1+x} dx$$

$$5. \int \frac{\ln^2 x}{x} dx$$

$$6. \int \frac{e^x}{e^x+1} dx$$

$$7. \int 3^{2x} dx$$

$$8. \int 8^{2x-1} dx$$

$$9. \int x 5^{x^2} dx$$

$$10. \int \frac{5 \csc \sqrt{x}}{\sqrt{x}} dx$$

$$11. \int \sqrt{x+3} (x+1)^2 dx$$

$$12. \int \frac{x^3}{\sqrt{1-2x^2}} dx$$

$$13. \int x^2 10^{x^3} dx$$

$$14. \int 5^{x^4+2x} (2x^3 + 1) dx$$

$$15. \int \ln e^{\sin x} dx$$

$$16. \int (3 \cos x) 8^{\sin x} dx$$

$$17. \int \frac{5x}{\sqrt{x-2}} dx$$

$$18. \int e^{8x-1} dx$$

$$19. \int \sec x \tan x \sqrt{1 + \sec x} \, dx$$

$$20. \int e^{\ln \csc x} \, dx$$

Differential Equations

Solve the following differential equations.

1. $\frac{dy}{dx} = y^2$

2. $y \frac{dy}{dx} = x$

3. $x^2 \frac{dy}{dx} + y = 0$

$$4. \frac{du}{dt} = e^{u+2t}$$

$$5. \frac{dy}{dx} = \frac{x^2 \sqrt{x^3 - 3}}{y^2}$$

$$6. \frac{dy}{dx} = \frac{\sqrt{x} + x}{\sqrt{y} - y}$$

7. $\frac{du}{dv} = \frac{\cos 2v}{\sin 3u}$

8. $\frac{d^2y}{dx^2} = 5x^2 + 1$

9. $\frac{dy}{dx} = (x + 1)(x + 2)$ and $y = -\frac{3}{2}$ when $x = -2$

10. $\frac{d^2y}{dx^2} = -\frac{3}{x^4}$ and $y = \frac{1}{2}$ and $\frac{dy}{dx} = -1$ when $x = 1$

11. Find the equation of the curve that satisfies $\frac{dy}{dx} = 4x^3y$ and whose y -intercept is 7.

12. Find the equation of the curve that passes through the point $(1, 1)$ and whose slope at any point (x, y) is given by $\frac{dy}{dx} = \frac{y^2}{x^3}$.

13. Given $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$ and $f(0) = \frac{1}{2}$, find $y = f(x)$.

14. Given $\frac{dy}{dx} = y^2(6 - 2x)$ and $f(3) = \frac{1}{4}$, find $y = f(x)$.

15. Given $\frac{dy}{dx} = -\frac{2x}{y}$ and $f(1) = -1$, find $y = f(x)$.

16. Given $\frac{dy}{dx} = \frac{1+y}{x}$ and $f(-1) = 1$, find $y = f(x)$.

17. Given $\frac{dy}{dx} = xy^3$ and $f(1) = 2$, find $y = f(x)$.

18. Given $\frac{dy}{dx} = \frac{y-1}{x^2}$ and $f(2) = 0$, find $y = f(x)$.