Advanced Placement Calculus

The Definite Integral

Sigma Notation
Approximating Area
Exact Area via Riemann Sums
The Definite Integral
Properties of the Definite Integral

Sigma Notation

1. Expand: $\sum_{k=1}^{4} \frac{1}{k}$

2. Expand: $\sum_{k=1}^{4} \frac{12}{k}$

3. Expand: $\sum_{i=1}^{3} (i+2)$

4. Expand: $\sum_{i=1}^{5} (2i+1)$

5. Expand: $\sum_{i=0}^{4} \frac{i}{4}$

6. Expand: $\sum_{k=-2}^{2} 3k$

7. Expand: $\sum_{k=1}^{4} \cos k\pi$

8. Expand: $\sum_{i=1}^{4} (-1)^i$

9. Expand: $\sum_{i=1}^{4} (-1)^{i+1}$

10. Express using sigma notation: $1 + 2 + 3 + 4 + 5 + 6$

11. Express using sigma notation: $1 + 4 + 9 + 16$

12. Express using sigma notation: $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$

13. Express using sigma notation: $\frac{1}{5} - \frac{2}{5} + \frac{3}{5} - \frac{4}{5} + \frac{5}{5}$

Problems 14-19: Evaluate the following sums. Use formulas when necessary.

14. $\sum_{k=1}^{10} k$

15. $\sum_{k=1}^{7} 2k$

16. $\sum_{i=1}^{6} \left(i^2 + 5\right)$

17. $\sum_{i=1}^{5} i(i-5)$

18. $\sum_{i=1}^{7} (2i-8)$

19. $\sum_{i=1}^{100} (2-5i)$

20. Evaluate: $\sum_{i=1}^{n} 2i$

21. Evaluate: $\sum_{i=1}^{n} (i^2+3i+4)$

22. Evaluate: $\sum_{i=1}^{n} (i+1)(i+2)$

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23. Evaluate: $\sum_{i=1}^{n} (i^3 - i - 2)$

Approximating Area

For each problem below approximate the area bounded by the curve and the x-axis using the method specified. Include a sketch of the area including the rectangles–lable the partition appropriately.

1. $f(x) = 6 - x^2$ from $x = 0$ to $x = 2$ using 5 rectangles and a left sum.

2. $f(x) = x^2 + 2$ from $x = -3$ to $x = 2$ using 5 rectangles and a right sum.

3. $f(x) = x^2 + 6$ from $x = 1$ to $x = 6$ using 5 rectangles and a midpoint sum.

4. $f(x) = x^2 - x + 3$ from $x = 0$ to $x = 3$ using 4 rectangles and a left sum.

5. $f(x) = x^2 - x + 3$ from $x = 0$ to $x = 3$ using 4 rectangles and a right sum.

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6. $f(x) = x^2 - x + 3$ from $x = 0$ to $x = 3$ using 4 rectangles and a midpoint sum.

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7. $f(x) = 2x^2 - 5x + 6$ from $x = -1$ to $x = 4$ using 5 rectangles and a left sum.

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8. $f(x) = 2x^2 - 5x + 6$ from $x = -1$ to $x = 4$ using 5 rectangles and a right sum.

9. $f(x) = 2x^2 - 5x + 6$ from $x = -1$ to $x = 4$ using 5 rectangles and a midpoint sum.

Exact Area

Determine the following areas using $A = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x$.

Note: Remember, since you are taking the limit of a Riemann sum, it will not matter whether you use a left, midpoint or right sum. Therefore, always use a right-sum, with $c_i = a + i\Delta x$.

1. The region bounded by $y = x^2$, the x-axis, from $x = 0$ to $x = 2$.

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2. The region bounded by $y = x^2$, the x-axis, from $x = 1$ to $x = 3$.

3. The region bounded by $y = 2x$, the x-axis, from $x = 1$ to $x = 4$.

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4. The region above the x-axis and to the right of $x = 1$ and the curve $y = 4 - x^2$.

5. The region bounded by $y = x^2 + 3x - 2$, the x-axis, from $x = 1$ to $x = 4$.

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6. The region bounded by $y = 2x^2 - 4x + 5$, the x-axis, from $x = -3$ to $x = 2$.

The Definite Integral

Evaluate the following definite integrals using the limit of a Riemann sum.

1. $\int_{1}^{3} (1 + 2x)\ dx$

2. $\int_{1}^{3} (x^2 + 4x)\ dx$

3. $\displaystyle\int_0^4 (x^2 + 2)\ dx$

Express each limit as a definite integral.

4. $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^4}{n^5}$

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5. $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(1 + \frac{3i}{n}\right) \frac{3}{n}$

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6. $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \left[3\left(1 + \frac{2i}{n}\right)^5 - 6\right] \frac{2}{n}$

Use the properties of definite integrals to express each as a single definite integral.

7. $\int_{1}^{3} f(x)\ dx + \int_{3}^{6} f(x)\ dx + \int_{6}^{12} f(x)\ dx$

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8. $\int_{5}^{8} f(x)\ dx + \int_{0}^{5} f(x)\ dx$

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9. $\int_{2}^{10} f(x)\ dx - \int_{2}^{7} f(x)\ dx$

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10. $\int_{-3}^{5} f(x)\ dx - \int_{-3}^{0} f(x)\ dx + \int_{5}^{6} f(x)\ dx$

11. Given $\int_0^6 h(x)\ dx = 15$ and $\int_0^4 h(x)\ dx = 3$, find $\int_4^6 [5h(x) - 7]\ dx$.

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12. Given $\int_1^7 p(x)\ dx = 20$ and $\int_7^{10} p(x)\ dx = 4$, find $\int_1^{10} [3p(x) - 2]\ dx$.

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13. Given $\int_2^5 f(x)\ dx = 3a - 2b$, find $\int_2^5 [4f(x) - 3]\ dx$.

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14. Given $\int_a^b g(x)\ dx = 3a + 11b$, find $\int_a^b [6g(x) + 6]\ dx$.

15. Given $\int\limits_a^b r(x)\ dx = 8a + 9b$, find $\int\limits_a^b [3r(x) - 7]\ dx$.

16. Given $\int\limits_a^b q(x)\ dx = a - 9b$, find $\int\limits_a^b [9q(x) - 1]\ dx$.

17. Given $\int\limits_3^7 f(x)\ dx = 20$ and $\int\limits_3^5 f(x)\ dx = 6a - 2b$, find $\int\limits_5^7 [4f(x) - 2]\ dx$.

18. Given $\int\limits_1^4 h(x)\ dx = 4a - 2b$ and $\int\limits_4^9 h(x)\ dx = 51$, find $\int\limits_1^9 [8h(x) - 5]\ dx$.

19. Given $\int_1^7 r(x)\ dx = 12$ and $\int_7^4 r(x)\ dx = -3$, find $\int_1^4 r(x)\ dx$.

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20. Given $\int_2^9 t(x)\ dx = 25$, and $\int_9^6 t(x)\ dx = -12$, find $\int_2^6 t(x)\ dx$.