

Advanced Placement Calculus

The Fundamental Theorems of Calculus

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Trapezoidal Rule

The Fundamental Theorems of Calculus–Part A

1. Evaluate: $\int_0^1 (2x - 1) \, dx$

2. Evaluate: $\int_0^1 (x^4 + x)(4x^3 + 1) \, dx$

3. Evaluate: $\int_1^2 x\sqrt{x-1} \, dx$

4. Evaluate: $\int_0^1 \cos \pi x \, dx$

5. Evaluate: $\int_1^4 \frac{1}{x^2} \sqrt{1 + \frac{1}{x}} dx$

6. Evaluate: $\int_0^{\pi/3} \frac{\sin x}{\cos^2 x} dx$

7. Evaluate: $\int_0^{13} \frac{1}{\sqrt[3]{(1+2x)^2}} dx$

8. Evaluate: $\int_0^4 \frac{dx}{(x-2)^3}$

9. Evaluate: $\int_0^a x\sqrt{x^2 + a^2} dx$, $a > 0$ (a is a constant)

10. Evaluate: $\int_{-2}^5 |x-3| dx$

11. Evaluate: $\int_0^1 \frac{x^3 - 1}{x + 1} dx$

12. Evaluate: $\int_0^{\pi/2} \sin 2x dx$

13. Evaluate: $\int_1^3 \frac{x}{(3x^2 - 1)^3} dx$

14. Evaluate: $\int_{-2}^1 (x+2)\sqrt{x+3} \, dx$

15. Evaluate: $\int_2^4 \frac{w^4 - w}{w^3} \, dw$

The Fundamental Theorems of Calculus–Part B

1. Evaluate: $\frac{d}{dx} \int_0^x \sqrt{4 + t^6} dt$

2. Evaluate: $\frac{d}{dx} \int_2^x \frac{1}{t^4 + 4} dt$

3. Evaluate: $\frac{d}{dx} \int_x^3 \sqrt{\sin t} dt$

4. Evaluate: $\frac{d}{dx} \int_{-x}^x \frac{1}{3 + t^2} dt$

5. Evaluate: $\frac{d}{dx} \int_1^{x^3} \sqrt[3]{t^2 + 1} dt$

6. Evaluate: $\frac{d}{dx} \int_2^{\tan x} \frac{1}{1 + t^2} dt$

7. Let $F(x) = \int_2^x \sqrt{3t^2 + 1} dt$. Find $F(2)$, $F'(2)$ and $F''(2)$.

8. Let $F(x) = \int_0^x \frac{\cos t}{t^2 + 3} dt$. Find $F(0)$, $F'(0)$ and $F''(0)$.

9. Evaluate: $\frac{d}{dx} \int_{2x}^{3x} \frac{u-1}{u+1} du$

10. Evaluate: $\frac{d}{dx} \int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}} dt$

11. Evaluate: $\frac{d}{dx} \int_{\sqrt{x}}^{x^3} \sqrt{t} dt$

12. Evaluate: $\frac{d}{dx} \int_{\cos x}^{5x} \cos u^2 du$

13. Given $F(x) + 3 = \int_5^x (t - 3)^3 \, dt$, find $F(5)$, $F'(5)$ and $F''(5)$.

14. Given $G(x) = \int_3^x \ln t \, dt$, find $G(3) - G'(3)$.

15. Let $F(x) + 7 = \int_8^x \sqrt{2t^2 - 5} \, dt$. Find $F(8)$, $F'(8)$ and $F''(8)$.

The Trapezoidal Rule

1. Evaluate $\int_0^2 x^3 \, dx$ using the trapezoid rule with $n = 4$.

2. Evaluate $\int_0^\pi \cos x \, dx$ using the trapezoid rule with $n = 4$.

3. Evaluate $\int_2^{10} \frac{dx}{1+x}$ using the trapezoid rule with $n = 8$.

4. Evaluate $\int_2^3 \sqrt{1+x^2} dx$ using the trapezoid rule with $n = 6$.