Advanced Placement Calculus

Area and Volume

Areas Between Curves
Volumes by Slicing
Volumes Using Dish/Washer Method
Volumes Using Shells

Areas Between Curves

1. Find the area of the region bounded by $y = x^2 + 3,\ y = x,\ x = -1$ and $x = 1$.

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2. Find the area of the region bounded by $x = y^3 - y$ and $x = 1 - y^4$.

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3. Find the area of the region bounded by $y = x$ and $y = x^2$.

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4. Find the area of the region bounded by $y = \sqrt{x}$ and $y = \dfrac{x}{2}$.

5. Find the area of the region bounded by $y = 4x^2$ and $y = x^2 + 3$.

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6. Find the area of the region bounded by $f(x) = x^2 + 2,\ g(x) = 2x + 5,\ x = 0$ and $x = 6$.

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7. Find the area of the region bounded by $y^2 = x$ and $x - 2y = 3$.

8. Find the area of the region bounded by $y = x,\ y = \sin x,\ x = -\dfrac{\pi}{4}$ and $x = \dfrac{\pi}{2}$.

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9. Find the area of the region bounded by $y = \cos x,\ y = \sin 2x,\ x = 0$ and $x = \dfrac{\pi}{2}$.

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10. Find the area of the region bounded by $x = 3y,\ x + y = 0$ and $7x + 3y = 24$.

11. Find the area of the region bounded by $y = x^2$ and $y = \dfrac{2}{x^2+1}$.

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12. Find the area of the region bounded by $y = 2^x,\ y = 5^x,\ x = -1$ and $x = 1$.

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13. Find the area of the region bounded by $f(x) = e^x,\ g(x) = e^{-x},\ x = -2$ and $x = 1$.

14. Find the area of the region bounded by $y^3 = x^2$ and $x - 3y + 4 = 0$.

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15. Find the area of the region bounded by $x = 4 - y^2$ and $x = 4 - 4y$.

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16. Find the area of the region bounded by $f(x) = 2x^3 - 3x^2 - 9x$ and $g(x) = x^3 - 2x^2 - 3x$.

1. Find the volume of the solid whose base is bounded by $x^2 + y^2 = 9$ and whose cross-section taken perpendicular to the x-axis are squares.

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2. Find the volume of the solid whose base is bounded by $y^2 = x - 5$ and $x = 10$ and whose cross-section taken perpendicular to the x-axis are equilateral triangles.

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3. Find the volume of the solid whose base is bounded by $y^2 = x - 5$ and $x = 10$ and whose cross-section taken perpendicular to the x-axis are squares.

4. Find the volume of the solid whose base is bounded by $y^2 = x - 5$ and $x = 10$ and whose cross-sections taken perpendicular to the x-axis are semicircles.

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5. Find the volume of the solid whose base is bounded by $y = x + 1$ and $y = x^2 - 1$ and whose cross-sections taken perpendicular to the x-axis are squares.

Problems 6-17: Use the disk/washer method. It is to your advantage to include a detailed and appropriately labeled sketch.

6. Find the volume of the solid generated when the region bounded by $y = x^2,\ x = 1$ and $y = 0$ is revolved about the x-axis.

7. Find the volume of the solid generated when the region bounded by $y^2 = x^3,\ x = 4$ and $y = 0$ is revolved about the x-axis.

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8. Find the volume of the solid generated when the region bounded by $y = x^2$ and $y^2 = x$ is revolved about the x-axis.

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9. Find the volume of the solid generated when the region bounded by $y = x^4$ and $y = 1$ is revolved about the line $y = 2$.

10. Find the volume of the solid generated when the region bounded by $y = 2x - x^2,\ y = 0,\ x = 0$ and $x = 1$ is revolved about the line $x = -3$.

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11. **Set up but do not evaluate** an integral which will yield the volume of the solid generated when the region bounded by $y = \ln x,\ y = 1$ and $x = 1$ is revolved about the x-axis.

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12. **Set up but do not evaluate** an integral which will yield the volume of the solid generated when the region bounded by $x - y = 1$ and $y = (x - 4)^2 + 1$ is revolved about the line $y = 7$.

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13. **Set up but do not evaluate** an integral which will yield the volume of the solid generated when the region bounded by $2x + 3y = 6$ and $(y - 1)^2 = 4 - x$ is revolved about the line $x = -5$.

14. Find the volume of the solid, in Quadrant I, generated when the region bounded by $y = \sqrt[3]{x},\ y = 2$ and $x = 0$ is revolved about the x-axis.

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15. Find the volume of the solid, in Quadrant I, generated when the region bounded by $y = \sqrt[3]{x},\ y = 2$ and $x = 0$ is revolved about the line $y = 2$.

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16. Find the volume of the solid, in Quadrant I, generated when the region bounded by $y = \sqrt[3]{x}$ and $x = 4y$ is revolved about the line $x = 8$.

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17. Find the volume of the solid, in Quadrant I, generated when the region bounded by $y = \sqrt[3]{x}$ and $x = 4y$ is revolved about the line $y = 2$.

Problems 18-29: Use the shell method. It is to your advantage to include a detailed and appropriately labeled sketch.

18. Find the volume of the solid generated when the region bounded by $y = x^2,\ y = 0,\ x = 1$ and $x = 2$ is revolved about the y-axis.

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19. Find the volume of the solid generated when the region bounded by $y = \dfrac{1}{x},\ y = 0,\ x = 1$ and $x = 10$ is revolved about the y-axis.

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20. Find the volume of the solid generated when the region bounded by $y = e^{-x^2},\ y = 0,\ x = 0$ and $x = 1$ is revolved about the y-axis.

21. Find the volume of the solid generated when the region bounded by $y = \sin x^2,\ y = 0,\ x = 0$ and $x = \pi$ is revolved about the y-axis.

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22. Find the volume of the solid generated when the region bounded by $y = x^2 - 6x + 10$ and $y = -x^2 + 6x - 6$ is revolved about the y-axis.

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23. Find the volume of the solid generated when the region bounded by $x = \sqrt[4]{y},\ x = 0$ and $y = 16$ is revolved about the x-axis.

24. Find the volume of the solid generated when the region bounded by $y = x^2,\ y = 0,\ x = 1$ and $x = 2$ is revolved about the line $x = 1$.

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25. Find the volume of the solid generated when the region bounded by $y = \sqrt{x-1},\ y = 0$ and $x = 5$ is revolved about the line $y = 3$.

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26. Find the volume of the solid generated when the region bounded by $y = 4x - x^2$ and $y = 8x - 2x^2$ is revolved about the line $x = -2$.

27. **Set up but do not evaluate** an integral which will yield the volume of the solid generated when the region bounded by $x = 4 - y^2$ and $x = 8 - 2y^2$ is revolved about the line $y = 5$.

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28. **Set up but do not evaluate** an integral which will yield the volume of the solid generated when the region bounded by $y = x^4$ and $y = \sin \dfrac{\pi x}{2}$ is revolved about the line $x = -1$.

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29. **Set up but do not evaluate** an integral which will yield the volume of the solid generated when the region bounded by $y = \dfrac{1}{1 + x^2}$, $y = 0$, $x = 0$ and $x = 3$ is revolved about the y-axis.