AP CALCULUS
DERIVATIVES TEST ADDITIONAL PROBLEMS

Differentiate the following. The ONLY simplification you should do is remove all negative and fractional exponents.

1. $f(x) = (2x^4 - x)\tan(x^2 - 1)$
2. $y = \sqrt[4]{\sin 5x}$
3. $y = (1 + \sec(5x + 1))^8$
4. $g(x) = \cot(5x + x^3)$
5. $f(x) = (\sqrt{x} + \sqrt[3]{x} + x^{-7})^5$
6. $y = (2 + \cos 7x)^3$
7. $y = \sqrt[3]{x}\cos 2x$
8. $h(x) = (x^3 + 6x^2)^7(9x + 1)^2$
9. $y = \dfrac{3x^5 + 2x^3 + 6x + 2}{x^3 + x}$
10. $f(x) = (4 + \sec(9x + 1))^7$
11. $y = \sqrt[3]{x^7}\cot\left(\dfrac{x}{3}\right)$
12. $g(x) = \sqrt{(2 - \cos 6x)^3}$
13. $y = \sqrt{x}\sin 2x$
14. $f(x) = \dfrac{\csc 3x}{x^2 + 2x}$
15. $h(x) = \left(\tan\dfrac{x}{3}\right)^2$
16. $y = \sqrt[3]{\cot 5x}$
17. $y = \dfrac{\sec 2x}{3x - x^2}$
18. $f(x) = \cot(3x^2 - x)$
19. $f(x) = (3 - \sin 7x)^4$
20. $y = (2x^4 - 3x)\tan(x^2 - 2x^3)$
21. $f(x) = (3 - \sin 5x)^4$
22. $y = \sqrt[5]{\tan 3x}$
23. $h(x) = \sqrt{2x - \sqrt[3]{x}}$
24. $y = \sin(3x^2 - 2x)$
25. $f(x) = \sqrt[4]{(3 + \sec 2x)^5}$
26. $y = \sqrt[3]{3x^3 + 5\sqrt{x}}$
27. $h(x) = \dfrac{3x^5 - 2x^3 + 6x + 2}{x^3 + 5x}$
28. $y = \dfrac{\sec 4x}{2x - x^3}$
29. $f(x) = \cot(4x^2 + 7)$
30. $y = \left(\cot\dfrac{x}{4}\right)^3$
31. $f(x) = \sqrt[3]{\sin 2x}$
32. $y = (\sqrt{x} + \sqrt[3]{x} + x^{-9})^5$
33. $h(x) = \cos(4x^2 + 7)$
34. $y = \sin(3x^2 - 2x)$
35. $f(x) = (2x - 7)^3(3x^4 - \sqrt{5x})$
36. $y = (1 + (1 + (1 + x^2)^2)^2)^2$

Find f' and f'' for each of the following functions. Use the same simplification rules as above.

1. $f(x) = \tan\dfrac{x}{4}$
2. $f(x) = (-3 + \cos x)^2$
3. $f(x) = \dfrac{x + 10}{x + 3}$
4. $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$
5. $f(x) = \sec 7x$
6. $f(x) = (6 - x)^7$
7. $f(x) = \sqrt{(3 - 2x)^5}$
8. $f(x) = 2x\sin 2x + \cos 2x$

Determine if the function is differentiable for all x.

1. $f(x) = \begin{cases} x+2 & \text{for } x \le -4 \\ -x-6 & \text{for } x > -4 \end{cases}$

2. $f(x) = \begin{cases} x & \text{for } x \le 0 \\ x^2 & \text{for } x > 0 \end{cases}$

3. $f(x) = \begin{cases} x^2-4 & \text{for } x < 2 \\ \sqrt{x-2} & \text{for } x \ge 2 \end{cases}$

4. $f(x) = \begin{cases} x^2 & \text{for } x < -1 \\ -1-2x & \text{for } x \ge -1 \end{cases}$

5. $f(x) = \begin{cases} 3x^2 & \text{for } x \le 2 \\ x^3 & \text{for } x > 2 \end{cases}$

6. $f(x) = \begin{cases} x^2+1 & \text{for } x < -1 \\ 1-x^2 & \text{for } x \ge -1 \end{cases}$

Solutions

1. $f'(x) = (2x^4 - x)[\sec^2(x^2-1)]2x + [\tan(x^2-1)][8x^3-1]$
2. $f'(x) = \dfrac{5\cos 5x}{4\sqrt[4]{\sin^3 5x}}$
3. $\dfrac{dy}{dx} = 8[1+\sec(5x+1)]^7[\sec(5x+1)\tan(5x+1)](5)$
4. $g'(x) = [-\csc^2(5x+x^3)][5+3x^2]$
5. $f'(x) = 5\left(\sqrt{x}+\sqrt[3]{x}+\dfrac{1}{x^7}\right)^4\left(\dfrac{1}{2\sqrt{x}}+\dfrac{1}{3\sqrt[3]{x^2}}-\dfrac{7}{x^8}\right)$
6. $\dfrac{dy}{dx} = 3(2+\cos 7x)^2(-\sin 7x)(7)$
7. $\dfrac{dy}{dx} = \left(\sqrt[3]{x}\right)(-\sin 2x)(2) + (\cos 2x)\left(\dfrac{1}{3\sqrt[3]{x^2}}\right)$
8. $h'(x) = (x^3+6x^2)^7(2)(9x+1)(9) + (9x+1)^7(7)(x^3+6x^2)^6(3x^2+12x)$
9. $\dfrac{dy}{dx} = \dfrac{(x^3+x)(15x^4+6x^2+6)-(3x^5+2x^3+6x+2)(3x^2+1)}{(x^3+x)^2}$
10. $f'(x) = 7\left[4+\sec(9x+1)\right]^6\left[\sec(9x+1)\tan(9x+1)\right](9)$
11. $\dfrac{dy}{dx} = \left(\sqrt[3]{x^7}\right)\left[-\csc^2\dfrac{1}{3}x\right]\left(\dfrac{1}{3}\right) + \left[\cot\dfrac{1}{3}x\right]\left[\dfrac{7}{3}\sqrt[3]{x^4}\right]$
12. $g'(x) = \dfrac{3}{2}\sqrt{2-\cos 6x}\,(\sin 6x)(6)$
13. $\dfrac{dy}{dx} = \left(\sqrt{x}\right)(\cos 2x)(2) + (\sin 2x)\left(\dfrac{1}{2\sqrt{x}}\right)$
14. $f'(x) = \dfrac{(x^2+2x)(-\csc 3x\cot 3x)(3)-(\csc 3x)(2x+2)}{(x^2+2x)^2}$
15. $h'(x) = \left(2\tan\dfrac{1}{3}x\right)\left(\sec^2\dfrac{1}{3}x\right)\left(\dfrac{1}{3}\right)$
16. $\dfrac{dy}{dx} = \dfrac{\left(-\csc^2 5x\right)(5)}{3\sqrt[3]{(\cot 5x)^2}}$
17. $\dfrac{dy}{dx} = \dfrac{(3x-x^2)(\sec 2x\tan 2x)(2)-(\sec 2x)(3-2x)}{(3x-x^2)^2}$
18. $f'(x) = \left[-\csc^2(3x^2-x)\right](6x-1)$
19. $f'(x) = 4(3-\sin 7x)^3(-\cos 7x)(7)$
20. $\dfrac{dy}{dx} = (2x^4-3x)\left[\sec^2(x^2-2x^3)\right](2x-6x^2) + \left[\tan(x^2-2x^3)\right](8x^3-3)$
21. $f'(x) = 4(3-\sin 5x)^3(-\cos 5x)(5)$

22. $\dfrac{dy}{dx} = \dfrac{(\sec^2 3x)(3)}{5\sqrt[5]{(\tan 3x)^4}}$

23. $h'(x) = \dfrac{2 - \dfrac{1}{3\sqrt[3]{x^2}}}{2\sqrt{2x - \sqrt[3]{x}}}$

24. $\dfrac{dy}{dx} = \left[\cos(3x^2 - 2x)\right](6x - 2)$

25. $f'(x) = \dfrac{5}{4}\sqrt[4]{3 + \sec 2x}\,(\sec 2x \tan 2x)(2)$

26. $\dfrac{dy}{dx} = \dfrac{9x^2 + \dfrac{5}{2\sqrt{x}}}{3\sqrt[3]{(3x^3 + 5\sqrt{x})^2}}$

27. $h'(x) = \dfrac{(x^3 + 5x)(15x^4 - 6x^2 + 6) - (3x^5 - 2x^3 + 6x + 2)(3x^2 + 5)}{(x^3 + 5x)^2}$

28. $\dfrac{dy}{dx} = \dfrac{(2x - x^3)(\sec 4x \tan 4x)(4) - (\sec 4x)(2 - 3x^2)}{(2x - x^3)^2}$

29. $f'(x) = [-\csc^2(4x^2 + 7)]\,(8x)$

30. $\dfrac{dy}{dx} = 3\left(\cot \dfrac{1}{4}x\right)^2 \left(-\csc^2 \dfrac{1}{4}x\right)\left(\dfrac{1}{4}\right)$

31. $f'(x) = \dfrac{(\cos 2x)(2)}{3\sqrt[3]{(\sin 2x)^2}}$

32. $f'(x) = 5\left(\sqrt{x} + \sqrt[3]{x} + \dfrac{1}{x^9}\right)^4 \left(\dfrac{1}{2\sqrt{x}} + \dfrac{1}{3\sqrt[3]{x^2}} - \dfrac{9}{x^{10}}\right)$

33. $h'(x) = [-\sin(4x^2 + 7)]\,(8x)$

34. $\dfrac{dy}{dx} = \left[\cos(3x^2 - 2x)\right](6x - 2)$

35. $f'(x) = (2x - 7)^3\left[12x^3 - \dfrac{5}{2\sqrt{5x}}\right] + \left(3x^4 - \sqrt{5x}\right)[3(2x - 7)^2]\,(2)$

36. $\dfrac{dy}{dx} = (2)(2)(2)(2x)(1 + x^2)(1 + (1 + x^2))^2(1 + (1 + (1 + x^2)^2)^2$ (Phew…if you didn't get this one…don't worry!

1. $f'(x) = \frac{1}{4}\sec^2 \frac{1}{4}x$

   $f''(x) = \frac{1}{4}\left[2\sec\frac{1}{4}x\right]\left[\sec\frac{1}{4}x\tan\frac{1}{4}x\right]\frac{1}{4}$

2. $f'(x) = (-2\sin x)(-3+\cos x)$

   $f''(x) = (-2\sin x)(-\sin x) + (-3+\cos x)(-2\cos x)$

3. $f'(x) = \frac{-7}{(x+3)^2}$

   $f''(x) = \frac{14}{(x+3)^3}$

4. $f'(x) = \frac{4x}{(x^2+1)^2}$

   $f''(x) = \frac{(x^2+1)(4)-(4x)2(x^2+1)(2x)}{(x^2+1)^4}$

5. $f'(x) = 7\sec 7x\tan 7x$

   $f''(x) = (7\sec 7x)(\sec^2 7x)(7) + (\tan 7x)7(\sec 7x\tan 7x)(7)$

6. $f'(x) = -7(6-x)^6$

   $f''(x) = -42(6-x)^5(-1)$

7. $f'(x) = -5\sqrt{(3-2x)^3}$

   $f''(x) = 15\sqrt{3-2x}$

8. $f'(x) = (2x)(\cos 2x)(2) + (\sin 2x)(2) - (\sin 2x)(2) = 4x\cos 2x$

   $f''(x) = (4x)(-\sin 2x)(2) + (\cos 2x)(4)$

1. $f'(x) = \begin{cases} 1 & \text{for } x < -4 \\ -1 & \text{for } x > -4 \end{cases}$

   f is not differentiable at $x = -4$ because $f'_+(-4) = -1$ but $f'_-(-4) = 1 \ \therefore \ f'(-4) \nexists$.

2. $f'(x) = \begin{cases} 1 & \text{for } x < 0 \\ 2x & \text{for } x > 0 \end{cases}$

   f is not differentiable at $x = 0$ because $f'_+(0) = 0$ but $f'_-(0) = 1 \ \therefore \ f'(0) \nexists$.

3. $f'(x) = \begin{cases} 2x & \text{for } x < 2 \\ \dfrac{1}{2\sqrt{x-2}} & \text{for } x > 2 \end{cases}$

   f is not differentiable at $x = 2$ because $f'_+(2) \nexists$.

4. $f'(x) = \begin{cases} 2x & \text{for } x < -1 \\ -2 & \text{for } x > -1 \end{cases}$

   <u>Continuity test at $x = -1$</u>
   $f(-1) = 1$
   $\lim\limits_{x \to -1^+} f(x) = 1$ and $\lim\limits_{x \to -1^-} f(x) = 1 \ \therefore \lim\limits_{x \to -1} f(x) = 1$
   f is continuous at $x = -1$ because $f(-1) = \lim\limits_{x \to -1} f(x)$
   f is differentiable at $x = -1$ because $f'_+(-1) = -2$ and $f'_-(-1) = -2 \ \therefore \ f'(-1) \exists$ and f is continuous at $x = -1$.

5. $f'(x) = \begin{cases} 6x & \text{for } x < 2 \\ 3x^2 & \text{for } x > 2 \end{cases}$

   <u>Continuity test at $x = 2$</u>
   $f(2) = 12$
   $\lim\limits_{x \to 2^+} f(x) = 8$ but $\lim\limits_{x \to 2^-} f(x) = 12 \ \therefore \lim\limits_{x \to 2} f(x) \nexists$
   f is not continuous at $x = 2$ because $\lim\limits_{x \to 2} f(x) \nexists$.
   f is not differentiable at $x = 2$ because f is not continuous at $x = 2$.

6. $f'(x) = \begin{cases} 2x & \text{for } x < -1 \\ -2x & \text{for } x > -1 \end{cases}$

   f is not differentiable at $x = -1$ because $f'_+(-1) = 2$ but $f'_-(-1) = -2 \ \therefore \ f'(-1) \nexists$.